The Game of Hex: The Hierarchical Approach

ABSTRACT. Hex is a beautiful and mind-challenging game with simple rules and a strategic complexity comparable to that of Chess and Go. Hex positions do not tend to decompose into sums of independent positions. Nevertheless, we demonstrate how to reduce evaluation of Hex positions to an analysis of a hierarchy of simpler positions. We explain how this approach is implemented in Hexy, the strongest Hex-playing computer program, and Gold Medalist of the 5th Computer Olympiad in London, August 2000.

1. Introduction

The rules of Hex are extremely simple. Nevertheless, Hex requires both deep strategic understanding and sharp tactical skills. The massive game-tree search techniques developed over the last 30–40 years mostly for Chess (Adelson-Velsky, Arlazarov, and Donskoy 1988; Marsland 1986), and successfully used for Checkers (Schaeffer et al. 1996), and a number of other games, become less useful for games with large branching factors like Hex and Go. For a classic 11×11 Hex board the average number of legal moves is about 100 (compare with 40 for Chess and 8 for Checkers).

Combinatorial (additive) Game Theory provides very powerful tools for analysis of sums of large numbers of relatively simple games (Conway 1976; Berlekamp, Conway, and Guy 1982; Nowakowski 1996), and can be also very useful in situations, when complex positions can be decomposed into sums of simpler ones. This method is particularly useful for an analysis of Go endgames (Berlekamp and Wolfe 1994; Müller 1999).

Hex positions do not tend to decompose into these types of sums. Nevertheless, many Hex positions can be considered as combinations of simpler subgames. We concentrate on the hierarchy of these subgames and define a set of deduction rules, which allow to calculate values of complex subgames recursively, starting from the simplest ones. Integrating the information about subgames of this hierarchy, we build a far-sighted evaluation function, foreseeing the potential of Hex positions many moves ahead.

In Section 2 we introduce the game of Hex and its history. In Section 3 we discuss the concept of virtual connections. In Section 4 we introduce the AND