

# On a decomposition method for finding winning strategy in Hex game

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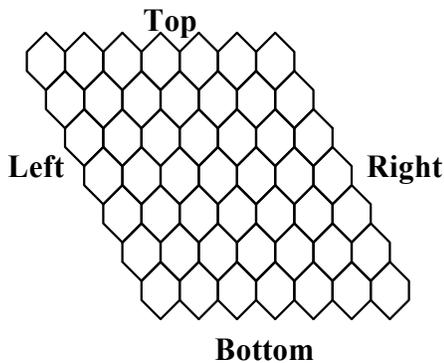
**Abstract--In this paper, a decomposition method is described that is very powerful on finding winning strategies in Hex game. A detail description of our solution for Hex on 7 by 7 board is also given. Up to now, 7 by 7 is the largest Hex board for which a winning strategy is actually known.**

**Index Terms--AI, Hex, winning strategy, decomposition method**

## I. INTRODUCTION

Hex is a board game in which two players play in turn and try to build a connected chain of pieces across opposite sides of the board. The Hex board is a hexagonal tiling of  $n$  rows and  $n$  columns. Usually  $n=11$  with 11 by 11 is the widely accepted standard board size. Figure 1 is an empty 7 by 7 Hex board.

**Figure 1: An empty Hex 7 by 7 board**



The rules of Hex game are simple:

- One player plays Black and the other plays White. Black owns the Top and Bottom sides and White owns the Left and Right sides.
- Black player plays first.
- Players take turns placing a piece of their color on an unoccupied hexagon.

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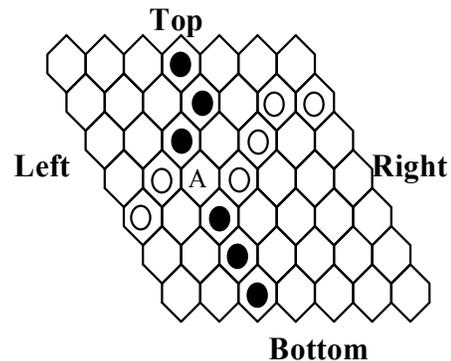
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- A game is won when one player establishes an unbroken chain of their pieces connecting their sides of the board.

For example, Figure 2 is in the middle stage of a Hex game and the next move is Black's turn. If Black plays the next move at position "A", Black will win the game. However, if Black plays at any other position rather than "A", White will play at position "A" and win the game.

**Figure 2: Take position "A" to win the game**



Hex was invented by Piet Hein in 1942. In 1949, John Nash proved that there is no tie in Hex game so that the first player has a theoretical win for Hex in any size. Later, Nash won Nobel Prize in Economics for his work in game theory. However, John Nash's proof is non-constructive. He didn't indicate how to win a Hex game. Over half a century, Hex only belongs to an ultra-weak solved game and a solution (winning strategy) for Hex has continued to elude researchers.

## II. THE APPROACH FOR FINDING SOLUTION

Hex is a NP-complete problem. It has a huge number of possible games when the size of the board is larger than or equal to seven. Fortunately, two important characteristics of Hex make it possible to find a winning solution on smaller size game. One is that Hex has a relatively smaller move selection space in most of situations comparing with several other board games such as Go and Chess. This characteristic is more obvious on smaller size Hex. For example, in Figure 3, if White doesn't play on "A", "B", "C", "D", "E", "F", "G", "H", and "I", Black will go "C" to win the game because three black stones (in a vertical line)